Energy transition meets mean-field games

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30th Birthday LMM, Université Le Mans, 23 May 2024

Outline

Climate change and energy transition

- The Paris Agreement is an international treaty on climate change. It was adopted by 196 Parties at the UN Climate Change Conference (COP21) in Paris, France, on 12 December 2015. It entered into force on 4 November 2016.
- Its overarching goal is to limit the temperature increase to 1.5°C by the end of this century.
- To limit global warming to 1.5°C, greenhouse gas emissions to be reduced by 45% by 2030 and reach net zero by 2050.

Climate change and energy transition

- Renewables, including solar, wind, hydropower, biofuels and others, are at the centre of the transition to less carbon-intensive and more sustainable energy systems. Increasing the supply of renewable energy would allow to replace carbon-intensive energy sources and significantly reduce global warming emissions (e.g., according to the U.S. Energy Administration website, 2022, about 31% of emissions of carbon dioxide (CO2) come from the electricity sector)
- In the Net Zero Emissions by 2050 scenario, renewables allow electricity generation to be almost completely decarbonised.

Climate change and energy transition

According to the International Energy Agency:

- Recent progress has been promising, and 2022 was a record year for renewable electricity capacity additions, with annual capacity additions amounting to about 340 GW.
- Solar PV is today the only renewable energy technology on track with the Net Zero Emissions by 2050 (NZE) Scenario. Wind, hydro, geothermal, solar thermal and ocean energy use needs to expand significantly faster in order to get on track.

Climate change and energy transition

Key strategies in the energy transition:

1. Ramping up renewables to displace fossil fuels
2. Demand side management $\rightarrow$ a duo of two approches: energy savings and demand response. The Energy Union refers to this as the "Energy Efficiency First Paradigm" (European Climate Foundation, 2016).

Mean-field games

- Introduced by Lasry and Lions (2006,2007) and Huang, Caines and Malhamé (2006) using PDE tools to describe large-population games with symmetric interactions in a tractable way
- Now 3000+ citations on Google Scholar, numerous applications in economics, finance, engineering, epidemiology etc.
- Mean-field games are a natural tool to describe interactions between different "actors" in the energy market!

Mean-field games

The MFG formulation

Define the asymptotic equilibrium state of the population as the solution of a fixed-point problem

- fix a flow of probability measures $(\mu_t)_{0 \le t \le T}$
- solve the stochastic optimal control problem in the environment $(\mu_t)_{0 \le t \le T}$

$$dX_t = \alpha_t dt + dW_t$$

  - With $X_0 = \xi$ being fixed on some probability space $(\Omega, \mathbb{F}, \mathbb{P})$ with a d-dimensional B.M.
  - With cost $J(\alpha) = \mathbb{E}\left[g(X_T, \mu_T) + \int_0^T (f(X_t, \mu_t) + \frac{|\alpha_t^2|}{2})\, dt\right]$

- let $(X_t^{\star,\mu})_{0 \le t \le T}$ be the unique optimizer (under nice assumptions) $\rightarrow$ find $(\mu_t)_{0 \le t \le T}$ such that

$$\mu_t = \mathcal{L}(X_t^{\star,\mu}).$$

Mean-field games

- Mean-field games with regular control have received a lot of attention, in terms of mathematical treatment and applications (e.g. Lasry-Lions papers, Carmona-Delarue, Bensoussan-Frehse-Yam)
- In contrast, optimal stopping mean-field games represent a new trend in the literature (e.g. Nutz, Carmona-Delarue-Lacker, Bertucci, Bouveret-Dumitrescu-Tankov, Dianetti-Ferrari-Fischer-Nendel..). For optimal stopping of McKean-Vlasov dynamics: Talbi-Touzi-Zhang.

Mean-field games

Approaches

- **PDE approach**: developed by Lasry and Lions (2006, 2007) and Huang, Malhamé and Caines (2006) $\rightarrow$ coupled system of partial differential equations: *Hamilton-Jacobi-Bellman* (backward) and *Fokker-Planck-Kolmogorov* (forward).
- **FBSDE approach**: introduced by Carmona and Delarue (2012) $\rightarrow$ *coupled forward-backward stochastic differential equations* with coefficients which depend on the law of the solution
- **Compactification methods**: Allow to solve the problem under mild assumptions by relaxing the concept of equilibrium.
  - *Controlled martingale approach* (introduced by Lacker (2015)).
  - *A linear programming approch* (introduced by Bouveret, Dumitrescu, Tankov (2020)).

Mean-field games

Back to the applications to electricity markets: who are the agents?

→ *Mean-field between producers*:
Djehiche, Barreiro-Gomez and Tembine (mean field game model for pricing electricity in a smart grid); Aïd, Dumitrescu, Tankov and Dumitrescu, Leutcher, Tankov (MFG to model exit of conventional producers and entry of renewables); Carmona,Dayanikli, Laurière (a mean-field model to regulate carbon emissions in electricity production)...

→ *Mean-field between consumers*: Bauso (dynamic demand and mean-field games), Alasseur, Matoussi, Ben Tahar and Alasseur, Campi, Dumitrescu, Zeng (interactive demand response on the consumers' side); Elie, Hubert, Mastrolia, and Possamaï (valuation of demand response contracts in a model with a continuum of consumers with mean field interactions); Alasseur, Bayraktar, Dumitrescu, and Jacquet (design of a new contract which rewards consumers depending on the rank of their consumption, which leads to competition between consumers)...

An entry-exit mean-field game for energy transition

Long-term model for the evolution of the structure of the electricity market

→ Aïd, Dumitrescu, and Tankov, "The entry and exit game in the electricity markets: a mean-field game approach." Journal of Dynamics and Games 8.4 (2021).

→ Dumitrescu, Leutscher, and Tankov, "Energy transition under scenario uncertainty: a mean-field game approach."Mathematics and Financial Economics (2024).

Objectives:

- **Our model:** Conventional producers aim to exit the market and renewable producers aim to enter the market. The producers interact through the electricity price. Study case on UK market.
- **Questions:**
  - → Which are the effects of this interaction and of the market mechanisms on the long-term price levels and the renewable penetration?
  - → Which are the impacts of the uncertainty on future carbon prices and of *scenario uncertainty* (determined by the uncertainty on future climate policies) on the dynamics of the electrical industry? In particular, at which rate conventional generation is replaced by renewable plants?
- **Mathematical resolution:** Mean-field games of optimal stopping (the linear programming approach)

Linear programming formulation for MFG

- A compactification technique, inspired by works on LP formulation of stochastic control (Stockbridge '90, Cho and Stockbridge '02).
- Related to the controlled martingale problem approach (El Karoui, Huu Nguyen, Jeanblanc '87), applied to MFG with regular controls in Lacker '15.
- Particularly suitable for MFG with optimal stopping / absorption: the lack of regularity of the flow μ_t makes it difficult to use the analytic approach.
- Applications to problems of investment timing, industry dynamics etc.

Linear programming formulation of MFG

Rationale and advantages

- Instead of iterating back and forth between the value function of the single agent and the population dynamics, the problem is formulated exclusively in terms of the population measure flow, which is the main object of interest.

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- The condition that the measure flow is the flow of marginal laws of a controlled stochastic process gives a linear constraint on the measure flow.

Linear programming formulation of MFG

Rationale and advantages

- Instead of iterating back and forth between the value function of the single agent and the population dynamics, the problem is formulated exclusively in terms of the population measure flow, which is the main object of interest.
- The condition that the measure flow is the flow of marginal laws of a controlled stochastic process gives a linear constraint on the measure flow.
- This formulation simplifies both the theoretical analysis of the problem (existence of equilibrium is established under weaker conditions) and the numerical computation of solutions.
- The "problem" of mixed controls is not a problem with LP formulation since different agents in a population can naturally use different strategies.
- Equivalence to "strong" formulations may be shown under appropriate assumptions.

Linear programming formulation for MFG

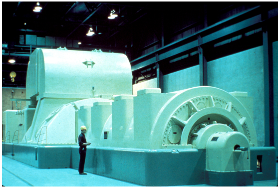
We develop theory and applications of the LP approach to MFG in a series of papers:

- Bouveret, Dumitrescu, and Tankov, "Mean-field games of optimal stopping: a relaxed solution approach." SIAM J. Con. Optim. 58.4 (2020).
- Aïd, Dumitrescu, and Tankov, "The entry and exit game in the electricity markets: a mean-field game approach." Journal of Dynamics and Games 8.4 (2021).
- Bouveret, Dumitrescu, and Tankov, "Technological Change in Water Use: A Mean-Field Game Approach to Optimal Investment Timing." Operations Research Perspectives 9 (2022).
- Dumitrescu, Leutscher, and Tankov, "Control and optimal stopping Mean Field Games: a linear programming approach." Electronic Journal of Probability 26 (2021).
- Dumitrescu, Leutscher, and Tankov, "Linear Programming Fictitious Play algorithm for Mean Field Games with optimal stopping and absorption." ESAIM: Mathematical Modelling and Numerical Analysis 57.2 (2023).
- Dumitrescu, Leutscher, and Tankov, "Energy transition under scenario uncertainty: a mean-field game approach," to appear in Mathematics and Financial Economics (2024).

Outline

The entry-exit game

The model: conventional producers



Each conventional producer has marginal cost function $C_t^i : [0,1] \to \mathbb{R}$: $C_t^i(\xi)$ is the unit cost of increasing capacity if operating at ξ

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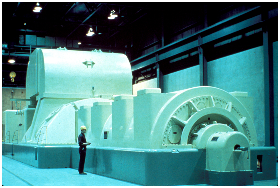

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We assume

$$C_t^i(\xi) = C_t^i + c(\xi)$$

where C_t^i is the baseline cost:

$$dC_t^i = k(\theta(t) - C_t^i)\,dt + \delta\sqrt{C_t^i}dW_t^i, \quad C_0^i = c_i,$$

and $c : \mathbb{R}_+ \to [0,1]$ is increasing smooth with $c(0) = 0$.

The entry-exit game

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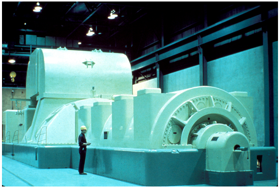

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The entry-exit game

The model: strategy of conventional producers

- For price p, conventional producer i chooses a proportion of capacity α of electricity production by maximizing

$$p\alpha - \int_0^\alpha C_t^i(y)\,dy.$$

  The optimal proportion of the capacity is $F(p - C_t^i)$, where $F = c^{-1}$.

- Gain of the producer at price level p is $G(p - C_t^i)$, where

$$G(x) = \int_0^x F(z)\,dz, \quad x \geq 0, \qquad G(x) = 0, \quad x < 0.$$

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- Each producer aims to exit the market at the optimal time τ_i given by

$$\max_{\tau} \mathbb{E}\left[\int_0^{\tau\wedge T} e^{-\rho t}(G(P_t - C_t^i) - \kappa_C)\,dt + K_C e^{-(\gamma_C+\rho)\tau\wedge T}\right],$$

  where P_t is the electricity price, K_C is the cost of assets recovered upon exit, κ_C is the fixed running cost and γ_C is the depreciation rate.

The entry-exit game

The model: renewable producers

Renewable producers aim to enter the market at the optimal time σ_i.

To enter they pay the cost K_R after which the plant generates $S_t^i \in (0,1)$ units of electricity per unit time at zero cost, where

$$dS_t^i = \bar{k}(\bar{\theta} - S_t^i)\,dt + \bar{\delta}\sqrt{S_t^i(1 - S_t^i)}\,d\overline{W}_t^i, \quad S_0^i = s_i \in (0,1).$$

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The renewable producers always bid their full intermittent capacity and solve:

$$\max_{\sigma_i} \mathbb{E}\left[\int_{\sigma_i \wedge T}^{T} e^{-\rho t}(P_t S_t^i - \kappa_R)\,dt - K_R e^{-\rho\sigma_i \wedge T} + K_R e^{-\rho T - \gamma_R(T - \sigma\wedge T)}\right],$$

where K_R is the fixed cost, κ_R is the running cost and γ_R is the depreciation rate.

The entry-exit game

The model: conventional supply. We denote by $\omega_t^n(dx)$ the distribution of costs of conventional producers who have not yet exited the market:

$$\omega_t^n(dx) = \frac{1}{n}\sum_{i=1}^{n} \delta_{C_t^i}(dx)\mathbf{1}_{\tau_i > t}.$$

The total supply by the conventional producers at price level p is

$$\int_0^T F(p-x)\omega_t^n(dx) + F_0(p),$$

where F_0 is the baseline supply.

The entry-exit game

The model: renewable supply

We denote by $\eta_t^n(dx)$ the (potential) distribution of output of renewable producers who have not yet entered the market and by $\bar{\eta}_t^n(dx)$ the distribution of output values of all renewable projects:

$$\eta_t(dx) = \frac{1}{n}\sum_{i=1}^{n} \delta_{S_t^i}(dx)\mathbf{1}_{\sigma_i > t}, \qquad \bar{\eta}_t^n(dx) = \frac{1}{n}\sum_{i=1}^{n} \delta_{S_t^i}(dx).$$

The total renewable supply at time t is given by $R_t^n = \int_0^1 x(\bar{\eta}_t^n(dx) - \eta_t^n(dx))$.

Entry-exit mean-field game

The model: Price formation. Agents are coupled through market price, by matching exogeneous demand process $\overline{D}_t$, to the aggregate supply function.

$$P_t := \inf\{P : (\overline{D}_t - R_t^n)^+ \leq \int_\Omega F(P-x)\omega_t^n(dx) + F_0(P)\} \wedge \overline{P},$$

where $\overline{P}$ is the price cap in the market.

When cap $\overline{P}$ is reached, demand is not entirely satisfied by producers.

Mean-field formulation

- To simplify the resolution and the study of price equilibria $\Rightarrow$ MFG framework, the number of agents is infinite.
- Limiting versions of the distributions ω_t^n, η_t^n and $\bar{\eta}_t^n$ are denoted by ω_t, η_t and $\bar{\eta}_t$, and the limiting market price and renewable demand by P_t and R_t.
- We assume first that the demand $\overline{D}_t$ is deterministic.
- Idiosyncratic noises of agents are independent and there is no common noise $\Rightarrow$ limiting measures are deterministic.

Mean-field formulation

State processes. Define the state processes of the representative agents:

$$dB_t = k(\theta - B_t)\,dt + \delta\sqrt{B_t}dW_t;\ \ B_0 = b_0,$$

and

$$dS_t = \bar{k}(\bar{\theta} - S_t)\,dt + \bar{\delta}\sqrt{S_t(1 - S_t)}\,d\overline{W}_t; \quad S_0 = s_0,$$

with $b_0 \sim \omega_0$ and $s_0 \sim \eta_0$, where $\omega_0 \in \mathcal{P}(\Omega)$ and $\eta_0 \in \mathcal{P}(\overline{\Omega})$. W and $\bar{W}$ are independent Brownians.

Mean-field formulation

LP formulation of the problem of conventional agents. Given a deterministic measurable price process $(P_t)_{t\geq 0}$, we replace the optimal stopping problem of individual conventional producer,

$$\sup_{\tau\in\mathcal{T}([0,T])}\mathbb{E}\left[\int_0^T e^{-\rho t}[G(P_t-B_t)+f_C(t)]\mathbf{1}_{t<\tau}\,dt\right],$$

by its LP version

$$\sup_{\omega\in\mathcal{A}(\omega_0)}\int_0^T\int_\Omega e^{-\rho t}[G(P_t-x)+f_C(t)]\omega_t(dx)\,dt,$$

where $\mathcal{A}(\omega_0)$ contains all flows of positive bounded measures $(\hat{\omega}_t)_{0\leq t\leq T}$ satisfying

$$\int_\Omega u(0,x)\omega_0(dx)+\int_0^T\int_\Omega\left\{\frac{\partial u}{\partial t}+\mathcal{L}u\right\}\hat{\omega}_t(dx)\,dt\geq 0$$

for all $u\geq 0, u\in C^{1,2}([0,T]\times\Omega)$ such that $\frac{\partial u}{\partial t}+\mathcal{L}u$ is bounded.

Mean-field formulation

LP formulation of the problem of renewable agents. Similarly, the optimal stopping problem of individual renewable producer

$$\sup_{\sigma \in \mathcal{T}([0,T])} \mathbb{E}\left[\int_0^T e^{-\rho t}\left\{-P_t S_t + f_R(t)\right\} \mathbf{1}_{t<\sigma}\, dt\right]$$

is replaced by its LP version

$$\sup_{\eta \in \overline{\mathcal{A}}(\eta_0)} \int_0^T \int_{\overline{\Omega}} e^{-\rho t}\left\{-P_t x + f_R(t)\right\} \eta_t(dx)\, dt,$$

where the set $\overline{\mathcal{A}}(\eta_0)$ is defined similarly to $\mathcal{A}(\omega_0)$.

Mean-field formulation

Price formation. Given the flows $(\omega_t)_{0\le t\le T}$ and $(\eta_t)_{0\le t\le T}$, the price $(P_t)_{0\le t\le T}$ is defined by

$$P_t = \inf\{P : (\overline{D}_t - R_t)^+ \le \int_\Omega F(P_t - x)\omega_t(dx) + F_0(P_t)\} \wedge \overline{P}, \quad 0 \le t \le T,$$

where $R_t = \int_{\overline{\Omega}} x(\bar{\eta}_t(dx) - \eta_t(dx))$. We denote the price by $P_t(\omega_t, \eta_t)$.

Mean-field formulation

Definition (LP MFG Nash equilibrium)

The LP Nash equilibrium is a couple (ω_t^*, η_t^*) such that for any other $\omega \in \mathcal{A}(\omega_0)$,

$$\int_0^T \int_\Omega e^{-\rho t}[G(P_t(\omega_t^*, \eta_t^*) - x) + f_C(t)]\omega_t(dx)\, dt \leq \int_0^T \int_\Omega e^{-\rho t}[G(P_t(\omega_t^*, \eta_t^*) - x) + f_C(t)]\omega_t^*(dx)\, dt,$$

and for any other $\eta \in \overline{\mathcal{A}}(\eta_0)$

$$\int_0^T \int_{\overline{\Omega}} e^{-\rho t}[-P_t(\omega_t^*, \eta_t^*)x + f_R(t)]\eta_t(dx)\, dt \leq \int_0^T \int_{\overline{\Omega}} e^{-\rho t}[-P_t(\omega_t^*, \eta_t^*)x + f_R(t)]\eta_t^*(dx)\, dt.$$

Mean-field formulation

Theorem

- *There exists a LP MFG Nash equilibrium problem.*
- *Let* (ω^1, η^1) *and* (ω^2, η^2) *be two Nash equilibria. Then, the set of points* t *such that* $P_t(\omega^1_t, \eta^1_t) \neq P_t(\omega^2_t, \eta^2_t)$ *has Lebesgue measure zero.*

Numerical algorithms

State of the art

- Several numerical algorithms have been proposed in the literature in the case of *regular control(without absorption)* , using analytic and probabilistic approaches (e.g. Achdou, Guéant, Laurière, Chassagneux, Crisan, Delarue). Another method, based on the fictitious play algorithm (learning procedure) has been introduced by Cardaliaguet-Hadikhanloo in the context of MFG of controls.
- Very few algorithms in the case of MFG of optimal stopping: Bouveret-D.-Tankov (potential games) and Bertucci (non-potential games, under a strict monotoniciy condition).

Numerical solution of the MFG problem

- The MFG problem is solved iteratively using the following algorithm
  - Choose starting point $m^{(0)} \in \mathcal{A}(m_0^*)$
  - For $n = 1, \ldots, N_{iter}$
    - Compute the best response

$$\tilde{m}^{(n)} = \arg \max_{m \in \mathcal{A}(m_0^*)} \int_0^T \int_\Omega f(t, x, m^{(n-1)}) m_t(dx)\, dt$$

    - Update the measure flow:

$$m^{(n)} = \frac{n-1}{n} m^{(n-1)} + \frac{1}{n} \tilde{m}^{(n)} = \frac{1}{n} \sum_{j=1}^{n} \tilde{m}^{(j)}.$$

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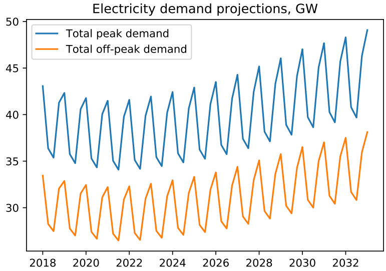
- To assess convergence, we monitor the “exploitability”:

$$\mathcal{E}(m^{(n)}) = \max_{m \in \mathcal{A}} \int_0^T \int_{\mathcal{O}} f(t, x, m^{(n)})(m_t - m_t^{(n)})(dx)\, dt$$

Empirically, we observe that $\mathcal{E}(m^{(n)}) \sim n^{-1}$ as $n \to \infty$.

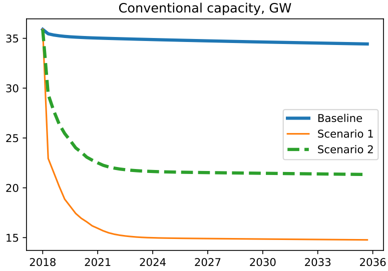
Numerical illustration: demand projections

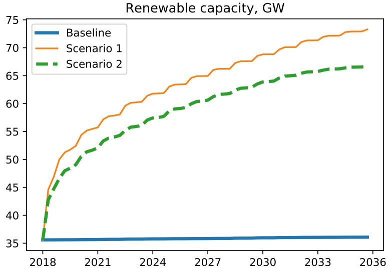


We distinguish peak / off-peak price/demand for more realistic projections

Demand projections from UK government forecast demand increase due to electrification

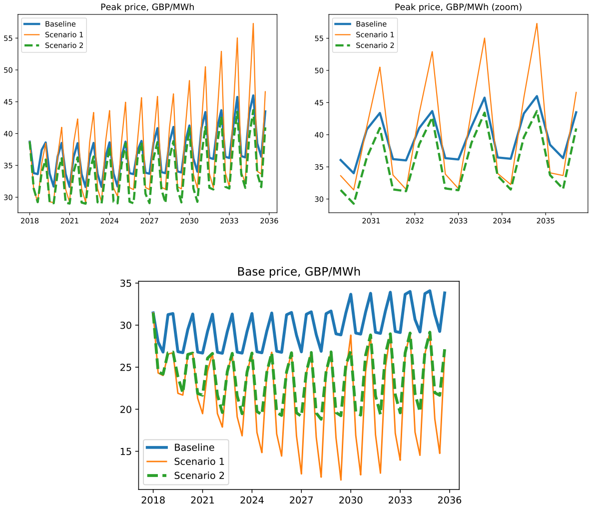
Numerical illustration: capacity





Conventional / renewable capacity evolution in three scenarios.
Baseline: costs estimated for UK market, no subsidy
Scenario 1: 30% renewable subsidy.
Scenario 2: renewable subsidy + a mechanism to keep conventional producers in the market.

Numerical illustration: price evolution



Outline

Model with common noise

- Conventional producers $\rightarrow$ Stochastic *baseline cost*.
  $\rightarrow$ Decide when to exit the market.
- Renewable producers $\rightarrow$ Stochastic *capacity factor*.
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- Carbon price (common noise) impacts the cost of the conventional producers and the demand.
- Supplies from conventional and renewable producers = Demand
- We look for *Nash equilibria*.

Model with common noise

- The demand process is

$$D_t = d(t) + \beta(Z_t - Z_0),$$

  $d(t)$ (in GW) deterministic function and $\beta \geq 0$.
  $\beta \geq 0$ (in GW×tonCO2/GBP): carbon price increases imply that carbon-intensive sectors of the industry are forced to electrify and contribute to electricity demand.

- The baseline cost of conventional producer i is

$$C_t^i(\alpha) = B_t^i + \tilde{\beta} Z_t + c(\alpha),$$

  where $\tilde{\beta} \geq 0$ (in tonCO2/MWh) represents the emission intensity.

Model with common noise

Carbon price

- Time period: 2022-2040. Time horizon $T_0 := 18$ years. *Discrete time model* with time steps of 3 months. We set $T = 4 \times 18 = 72$ and $I := \{0, 1, \ldots, T\}$ the set of time indices. We also set $\Delta t := T_0 / T = 0.25$.

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- The carbon price is denoted by $Z = (Z_t)_{t \in I}$ (GBP/tonCO2) and takes values in the finite set $H := \{50, 75, 100, 125, 150, 175, 200\}$. $Z_0 = 50$ GBP/tonCO2.

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- Total number of 64 possible trajectories of the carbon price Z accounting for possible jumps at some specific dates.

Numerical illustration

- We select the lowest trajectory of Z, denoted by ω, to plot the quantities of interest conditionally on Z taking this path.
- In addition, we plot the expectation (over the law of Z) of the quantities of interest.
- For comparison, we also solve the MFG model without common noise where the carbon price follows the trajectory ω with probability 1.

Numerical results

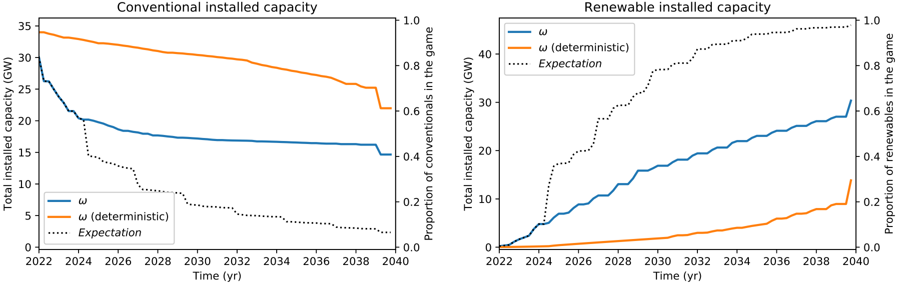

Figure: Total installed capacities and proportions of conventional and renewable producers.

Numerical results

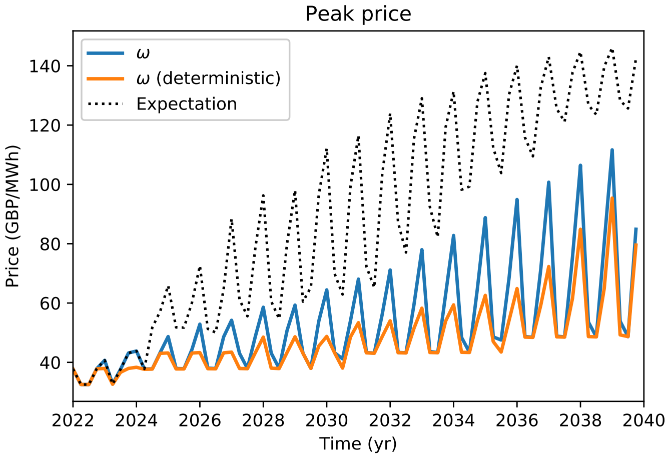

Figure: Peak prices.

Happy 30th Birthday to LMM!